

Reliability-Based Load-Rating of Existing Bridges

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SYNOPSIS

Work has been carried out to develop theoretical (probabilistic) models of loads and resistances, and to develop calculation procedures to provide a general framework for the evaluation of the reliability of existing bridges. A probabilistic model of peak truck loads has been developed to model not only the anticipated truck loads (accounting for the expected number of over-load events), but also historical loads (which act as uncertain proof-loads). Reliability analyses have been carried out considering combinations of uncertain resistances (such as ultimate moment capacities) together with uncertain load-effects due to the permanent loading (the weight of the bridge) and traffic loading. For the purposes of developing a simple checking procedure, reliability results have been evaluated in relation to the uncertain reserve resistance (i.e., the resistance that is available to resist the traffic loading after the application of the permanent loads). Results have also been obtained to illustrate the enhancement of reliability after a period of trouble-free traffic loading (constituting an uncertain proof-load).

The probabilistic model of truck loads is outlined in the paper. General reliability results are presented in the form of reliability contours on graphs based on the mean and coefficient of variation of the reserve strength (estimated by the design engineer). Reliability contours are given for structures with or without a history of previous traffic-loading. A simple procedure to check the structural reliability for a specified level of traffic loading is described and illustrated. Sample reliability calculations are also presented for a particular bridge on Camp St in the Municipality of Forbes.

1 PROPOSED METHODOLOGY FOR RELIABILITY-BASED LOAD-RATING

In 2003, the Roads and Traffic Authority of New South Wales (RTA) commissioned the Centre for Advanced Structural Engineering (CASE) at the University of Sydney, to develop a procedure for the reliability-based load-rating of existing bridges (particularly for old bridges which don't satisfy current design standards). As a result, a simple methodology was proposed for routine application by RTA engineers, based on the following steps (for critical structural elements and critical limit states):

- i. Assess the nominal bridge resistance (or strength) R_{nom} using standard (codified) procedures and standard strength equations for the relevant limit state (based on the Code strength equations for bending, shear, torsion, etc)
- ii. Estimate the actual (uncertain) strength limit R (expected value and coefficient of variation), based on the nominal resistance R_{nom} and relevant values of the mean bias factor and coefficient of variation associated with the standard strength equations used to calculate R_{nom}
- iii. Estimate the actual (uncertain) dead load and relevant dead load effect D (expected value and coefficient of variation)
- iv. Estimate the reserve strength R^* (expected value, variance and coefficient of variation), based on the statistics of the estimated strength R and dead load effect D , where $R^* = R - D$
- v. Assess the nominal traffic load effect T_{nom} corresponding to the legal traffic load limit

- vi. Divide the expected value of the reserve strength by the nominal traffic load effect to obtain a normalised expected value of the reserve strength
- vii. Determine the reliability index β from a relevant graph showing contours of β as a function of the normalised expected value of the reserve strength and the coefficient of variation of the reserve strength. (Note that the reliability contours also depend on the relevant design life and the extent of prior loading.)

Details of the probabilistic modelling of the load-effects and resistances are described below. The general basis of the reliability calculations is described and some reliability results are presented. The application of the procedure is demonstrated with a simple example based on the assessment of an existing bridge.

The aim of the work to date has been to demonstrate a feasible approach to reliability-based load-rating of existing bridges. Further work is required to develop procedures that can be implemented in practice, and the extension of the work is briefly discussed.

2 PROBABILISTIC MODEL OF TRUCK LOADS

In order to model truck loads, accounting for the period and frequency of loading (anticipated or historical), a new probabilistic model of truck loads has been developed as follows.

The truck loads are modelled with regard only to loads greater than or equal to the legal load limit. Thus the distribution of truck loads is modelled with regard to overload events. Assuming that overload events occur at a mean frequency ν over a time interval τ , then the expected number of overload events in that time interval is simply $n = \nu\tau$.

It is assumed that there is a physical upper-bound on the magnitude of an overload event. Considering normalised traffic loads s such that the legal limit corresponds to a load value of unity, the upper limit on normalised traffic loads is set equal to s_{max} . Accordingly the range of normalised overloads is $1.0 \leq s \leq s_{max}$.

The probability that the magnitude of an overload event exceeds a value s ranges from 1 when $s=1.0$ to 0 when $s = s_{max}$. Accordingly, the distribution function of overload magnitudes is expressed:

$$F(u) = (u/u_{max})^m \quad (1)$$

$$\text{where } u = (s_{max} - s) \quad (2)$$

$$u_{max} = (s_{max} - 1) \quad (3)$$

and $F_U(u)$ is the Cumulative Distribution Function (CDF) of u . Accordingly, the exponent m characterises the shape (or spread) of the distribution function $F_U(u)$. Considering a ‘‘characteristic value’’ of overloading of 25%, if the characteristic value corresponds to the 95th (or 99th) percentile of overloaded trucks then m takes the value 16.43 (or 25).

Assuming overload events are Poisson-distributed (in time), the Cumulative Distribution Function (CDF) of the maximum overload magnitude s (over a period τ) may be expressed:

$$F_S(s) = \exp[-\nu\tau(1-s/u_{max})^m] \quad (4)$$

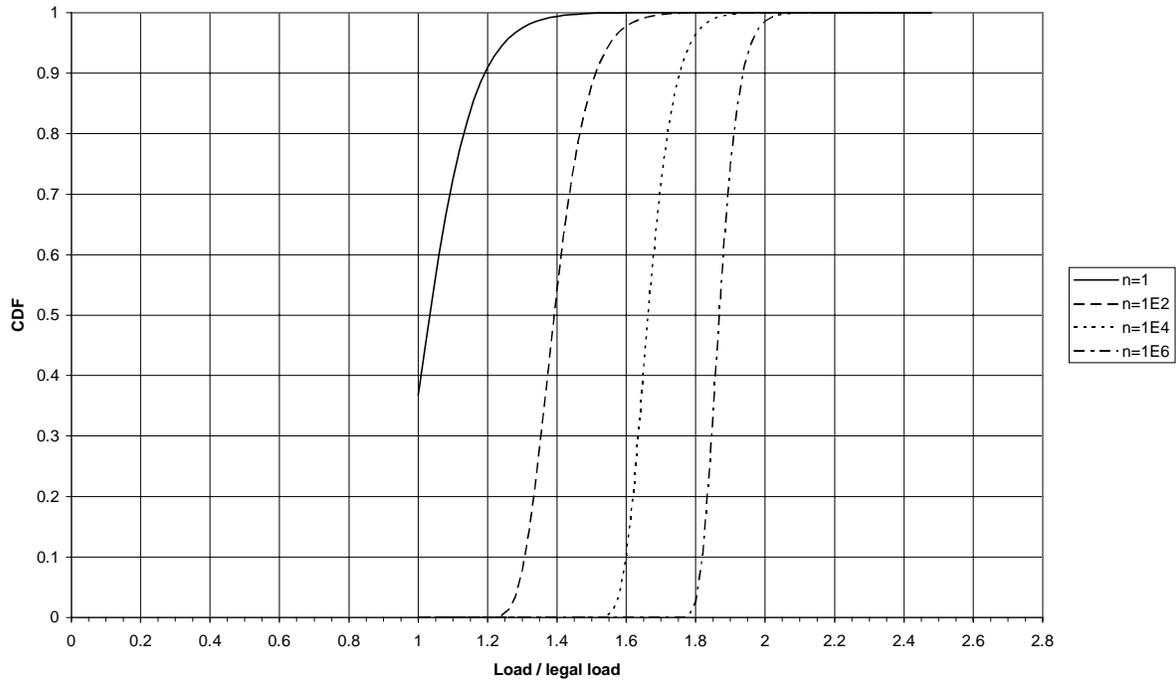


Figure 1: Cumulative Distribution Functions of peak truck loads – STXX(2.5,16.43,n)

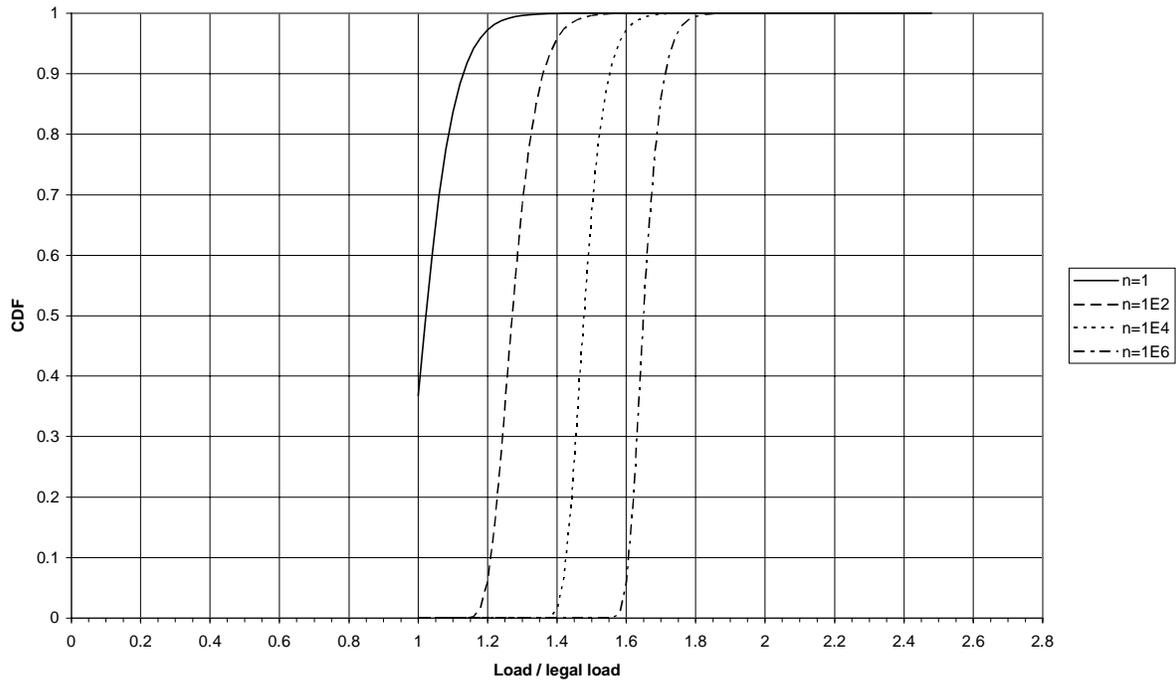


Figure 2: Cumulative Distribution Functions of peak truck loads – STXX(2.5,25,n)

The function $F_S(s)$ is used to model the CDF of the peak truck load, denoted $STXX(s_{max}, m, n)$, where ST denotes a standard 6-axle articulated truck and XX denotes the nominal gross weight in tonnes. CDFs of peak truck loads are shown in Figure 1 and Figure 2, considering particular values of the expected number of overload events n ranging from 1 to 10^6 (1E6). Figure 1 shows traffic load distributions for a characteristic 25% overload with a characteristic overload percentile of 95%

($m=16.43$) and Figure 2 shows traffic load distributions corresponding to a characteristic overload percentile of 99% ($m=25$).

3 RELIABILITY ASSESSMENT

Analyses have been carried out to assess the reliability of structures (or structural elements) subject to truck loading, based on the expected value and Coefficient of Variation (CoV) of the normalised reserve strength and the required design life (i.e., the expected number of overload events). The structural reliability was assessed using an Advanced First Order Second Moment method of analysis.

Typical reliability results are presented in Figure 3 which shows contours of the “safety index” β , based on the estimated mean and CoV of the reserve strength (normalised with respect to the value of the load-effect corresponding to the legal load limit). Figure 3 shows results for a LogNormal distribution of reserve strengths and a peak truck load distribution $STXX(2.5,25,1E6)$ as shown in Figure 2 (for an expected number of overload events $n=10^6$). Reliability contours are given in the region of $\beta=3$ (a common target value) for coefficients of variation V_R in the region of 0.1-0.3 (typical of normal structural strengths) and values of $MR/STXX$ (i.e., the expected reserve resistance divided by the legal load-effect) in the region 2-3.5.

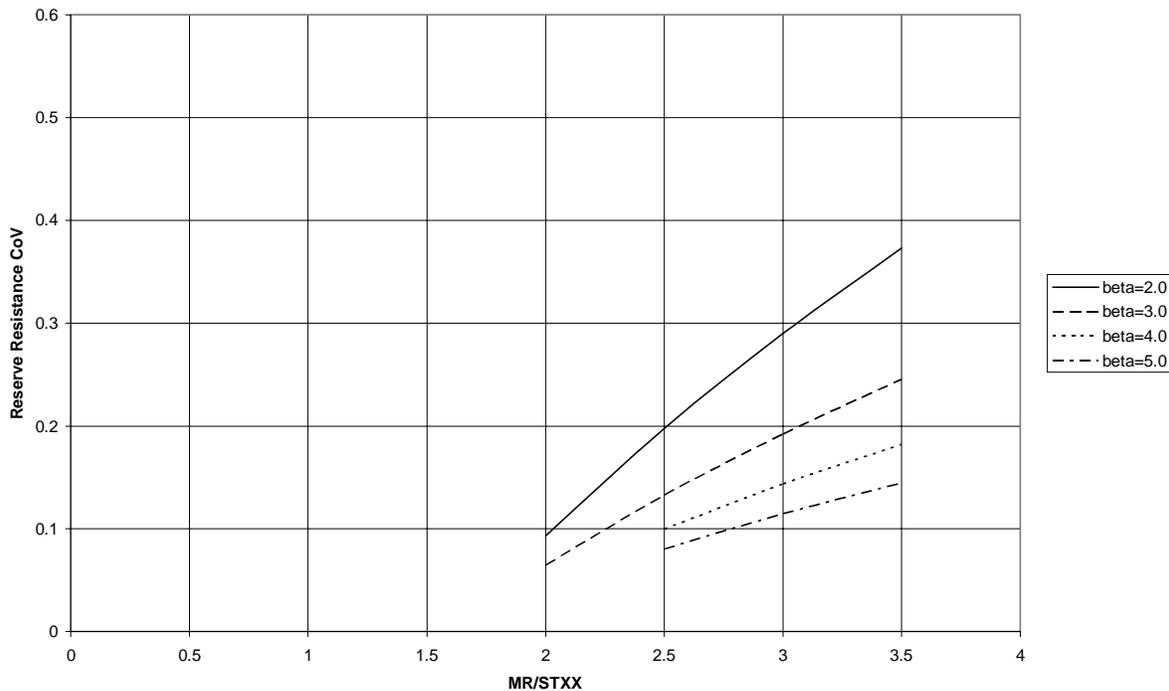


Figure 3: Contours of the Reliability Index β for a LogNormal reserve resistance and truck loads $STXX(2.5,25,1E6)$

3.1 Reliability with Historical (Uncertain) Proof-Loading

Many existing bridges have a significant history of trouble-free operation under traffic loading. The in-service traffic loading constitutes an uncertain proof-load, and it enhances the estimates of strength and reliability. The distribution of prior resistances R (prior to proof-loading) is

transformed into a distribution of posterior resistances R' (after proof-loading), depending on the probability distribution of uncertain proof-loads S :

$$F_{R'}(r) = F_S(r) - (1 - F_R(r))I(r) \quad (5)$$

$$f_{R'}(r) = f_R(r)I(r) \quad (6)$$

where, $I(r) = \int_0^r \frac{f_S(x)}{1 - F_R(x)} dx$ (7)

and where $f_X(x)$ and $F_X(x)$ denote the probability density and cumulative distribution functions, respectively.

Reliability analyses have been carried out to assess the effects of uncertain in-service proof-loading, and typical results after proof-loading are shown in Figure 4. The results shown in Figure 4 differ from the results shown in Figure 3 only because they are based on a resistance distribution that has been up-dated based on a history of traffic loading (uncertain proof-loading) of the same type as the projected loading (viz. STXX(2.5,25,1E6)). Updating of this type could be relevant to the assessment of safety for a bridge that doesn't satisfy current design standards.

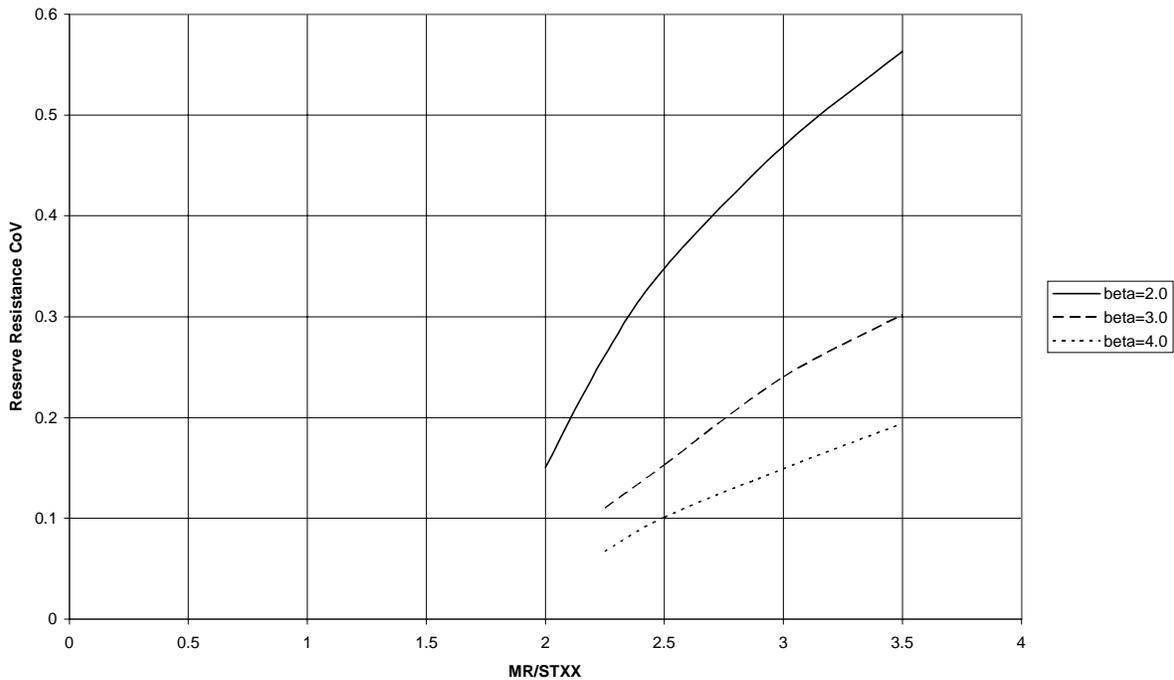


Figure 4: Contours of the Reliability Index β for a LogNormal reserve resistance and truck loads STXX(2.5,25,1E6) after in-service proof-loading (STXX(2.5,25,1E6))

A comparison of the results presented in Figure 3 and Figure 4 shows that the proof-loading enhances the calculated reliability, particularly for relatively high values of V_R and relatively low values of β .

4 EXAMPLE: Reliability-Based Load-Rating Assessment of Bridge on Camp St (Municipality of Forbes)

The reliability-based load-rating procedure proposed for routine application by bridge engineers (based on the methodology described above) simply involves estimating the expected value and Coefficient of Variation of the reserve strength (considering all relevant modes of behaviour) and determining the corresponding reliability (for a selected level of traffic loading) using normalised results similar to those shown in Figure 3 and Figure 4.

The general procedure is illustrated in the following example including sample (approximate) calculations for a reliability-based load-rating assessment of an existing bridge on Camp St in the Municipality of Forbes.

The approximate bridge dimensions are:

- maximum span: 7.215 m
- reinforced concrete girders (x 5): 710 mm x 280 mm and
- bridge deck: thickness 180 mm; width 9.15 m.

The analytical load-rating of the bridge is ST39 (for a 6 axle articulated truck of 39 tonnes), based on the mid-span bending strength of the girders. However, an increased load-rating of ST42.5 is proposed. The proposed load-rating is assessed using the reliability-based assessment procedure, as described below (based on the mid-span bending strength of the girders).

Step 1: Assess the nominal bending strength of the critical girder

The nominal strength should be assessed in accordance with the relevant design code, using the normal strength equations (in this case, for M_{uo}) based on the member geometry, reinforcement and material strengths. The nominal bending strength of a typical girder is estimated to be:

- $M_{uo} = 521 \text{ kNm}$.

(This estimate of the ultimate moment capacity M_{uo} has been inferred from the analytical load rating, assuming ST39 loading is effectively uniformly distributed over the span, with load-sharing for 3 girders, with a dead load factor $\gamma_G=1.2$ and a traffic load factor $\gamma_{\text{traffic}}=2.6$, including the dynamic load factor.)

Step 2: Estimate the actual strength (expected value and coefficient of variation)

The expected value of the actual strength (M_u) is approximately 10% greater than the nominal value (M_{uo}) and the Coefficient of Variation is approximately 14% (Ellingwood et al (1)). (In future work, tables of relevant factors will be prepared to assist design engineers.) Hence, the estimated strength statistics are:

- $E[M_u]=(1.10)(521)=573 \text{ kNm}$ and
- $\sigma_{M_u}=(0.14)(573)=80 \text{ kNm}$.

Step 3: Estimate the actual dead load effect (expected value and coefficient of variation)

The nominal dead load acting on a critical girder (including 50 mm topping) is estimated to be 14.3 kN/m. The expected value of the dead load is taken equal to the nominal value (based on actual dimensions) and the estimated coefficient of variation is 5%. Hence the statistics of the applied bending moment due to the dead load at midspan are:

- $E[M_D]=(14.3)(7.215)^2/8=93.1 \text{ kNm}$ and
- $\sigma_{M_D}=(0.05)(93.1)=4.7 \text{ kNm}$

Step 4: Estimate the reserve strength (expected value and coefficient of variation)

The reserve strength R^* is the actual strength minus the strength required to resist the dead loads. The statistics of the reserve strength are determined from the estimated statistics of the actual strength and the actual dead load effect:

- $E[R^*]=E[M_u]-E[M_D]=573-93.1=480 \text{ kNm}$ and
- $\sigma_{R^*}=[(\sigma_{Mu})^2+(\sigma_{MD})^2]^{1/2}=[80^2+4.7^2]^{1/2}=80.1 \text{ kNm}$
- Coefficient of Variation $V_{R^*}=\sigma_{R^*}/E[R^*]=80.1/480=0.167$

Step 5: Assess the traffic load effect (ST42.5)

Assuming the ST42.5 loading is effectively uniformly distributed over the span, with load-sharing for 3 girders and a dynamic load factor $\gamma_{dynamic}=1.3$, the corresponding midspan bending moment in a critical girder (corresponding to the legal traffic load limit) is:

- $M_{ST42.5}^*=166 \text{ kNm}$

Step 6: Normalise the reserve strength (with respect to the legal load effect)

The statistics of the normalised reserve strength are obtained from the statistics of the reserve strength (Step 4) by dividing by the legal traffic load effect (Step 5):

- $E[R^\#]=E[R^*]/M_{ST42.5}^*=480/166=2.89$
- $\sigma_{R^\#}=\sigma_{R^*}/M_{ST42.5}^*=80.1/166=0.483$
- Coefficient of Variation $V_{R^\#}=\sigma_{R^\#}/E[R^\#]=0.483/2.89=0.167$

Step 7: Determine the Reliability Index using standardised results based on statistics of the normalised reserve strength

For the specified truck loading and resistance type (with or without a load history) refer to standardised results to determine the Reliability Index β (also known as the Safety Index).

For example, for truck loads STXX(2.5,25,1E6) and a LogNormal reserve resistance (with no loading history) refer to Figure 3 to determine the Reliability Index corresponding to the expected value $E[R^\#]$ and the Coefficient of variation $V_{R^\#}$ of the normalised reserve strength (from Step 6). In this case ($E[R^\#]=2.89$ and $V_{R^\#}=0.167$), and the indicated result is:

- Reliability Index $\beta=3.3$.

Similarly, for truck loads STXX(2.5,25,1E6) and a LogNormal reserve resistance with a loading history of the same type as the anticipated loading, refer to Figure 4 to determine the Reliability Index. In this case ($E[R^\#]=2.89$ and $V_{R^\#}=0.167$), and the indicated result is:

- Reliability Index $\beta=3.6$.

5 FURTHER WORK

Work is continuing to further develop the methodology outlined above and to develop practical implementation aids and extend the range of normalised results that can be referred to by bridge engineers. In particular, further work is being carried out to complete the following tasks:

- preparation of Tables of estimated coefficients of variation and mean bias factors for standard strength equations
- preparation of a Table of estimated coefficients of variation of Dead Load for standard bridge types (and sizes)

- calibration of the traffic load model $STXX(s_{max},m,n)$ to match available data (for peak load effects due to a single vehicle)
- validation of the LogNormal reserve strength model (involving further probability studies)
- determination of further reliability results based on the truck load model $STXX$ and accounting for a range of load-histories (proof-loading), and
- development of a spreadsheet for simplified assessment of reliability (as an alternative to the chart-based approach described above).

The initial work outlined in this paper concentrated on the assessment of load-effects associated with a single vehicle, using the peak load model $STXX$. Further work is being carried out to obtain equivalent results for load-effects associated with load combinations involving multiple vehicles.

6 REFERENCES

- 1 ELLINGWOOD B., et al. "Development of a Probability Based Load Criterion for American National Standard A58", *National Bureau of Standards Special Publication 577*, U.S. Dept of Commerce, June, 1980.

7 ACKNOWLEDGEMENT

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