

Reliability Analysis to Verify the Currently used Partial Safety Factors in Bridge Design: A Case Study using Baandee Lakes Bridge No. 1049

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SYNOPSIS

Reinforced concrete flat slab bridges form a significant proportion of the older bridges in Western Australia. Many of these bridges are found to be “deficient” in load capacity when assessed by currently used elastic methods, but can be justified to have a higher load capacity using the yield line analysis method. A load rating increase using any method will inevitably lead to reduction in the safety levels previously enjoyed. The question is whether the reduced safety levels are still satisfactory.

This paper presents some of the results from a research program to investigate the suitability of the currently used partial safety factors with the yield line approach. The safety factors currently used in the Austroads Bridge Design Code (ABDC) are mainly based on past-experience. Changing the analysis method from elastic to yield line analysis, therefore require an appraisal of the safety factors used. To carry out this task, a reliability analysis was carried out based on the guidance provided by Eurocode. The analysis was undertaken by using a commercially available software, COBRAS.

The paper presents (a) a critical review of the reliability analysis methodology adopted in the software COBRAS, (b) a methodology for a reliability analysis, (c) the results of the reliability analysis of Bridge No. 1049, and (d) conclusions about the existing partial safety factors used in Austroads design guides.

1 INTRODUCTION

Elastic method of load rating bridge limits the capacity of the bridge to the load when a bending moment reaches the moment capacity of the slab at any point in the slab. This is very conservative. Yield line analysis allows hinges to form and the maximum load is determined by the mechanism. The yield line method would always provide a higher load rating than the elastic method. The difference between the load ratings will vary depending on the reinforcement and slab configurations.

Increasing the load ratings using any justification or rationale will inevitably lead to reduction in safety levels previously enjoyed. The question is whether the reduced safety levels are still satisfactory.

The safety factors currently used in standards and codes are mainly based on experience. It is not known how much of the existing safety levels come from the use of conservative analytical methods (such as elastic methods), and how much from the actual safety factors

used. Changing the analysis method from elastic to yield line analysis, therefore require an appraisal of the safety factors used. Some early discussions on these issues were presented by Leicester et al (9).

This paper presents a case study of the reliability analysis performed on Bridge No. 1049 and an appraisal of the currently used safety factors. The bridge was load-rated using yield line analysis and was later destructively tested to verify the load rating. The details of the bridge are presented in a companion paper (13).

The reliability analysis presented in this paper follows the guidance provided by the Eurocode – Basis of Structural Design, EN 1990, published in 2002 (2). The code provides a number of different guidelines for calibrating partial safety factors.

2 METHOD FOR INDEPENDENTLY ASSESSING LOAD AND RESISTANCE EFFECTS

In Annex C, Eurocode (2) provides guidance for independently assessing safety factors for load and resistance effects. Design values should be based on the values of the basic variables at the FORM (First Order Reliability Method) design point, which can be defined as the point on the failure surface closest to the average point in the space of normalised variables, as diagrammatically indicated in Figure 1.

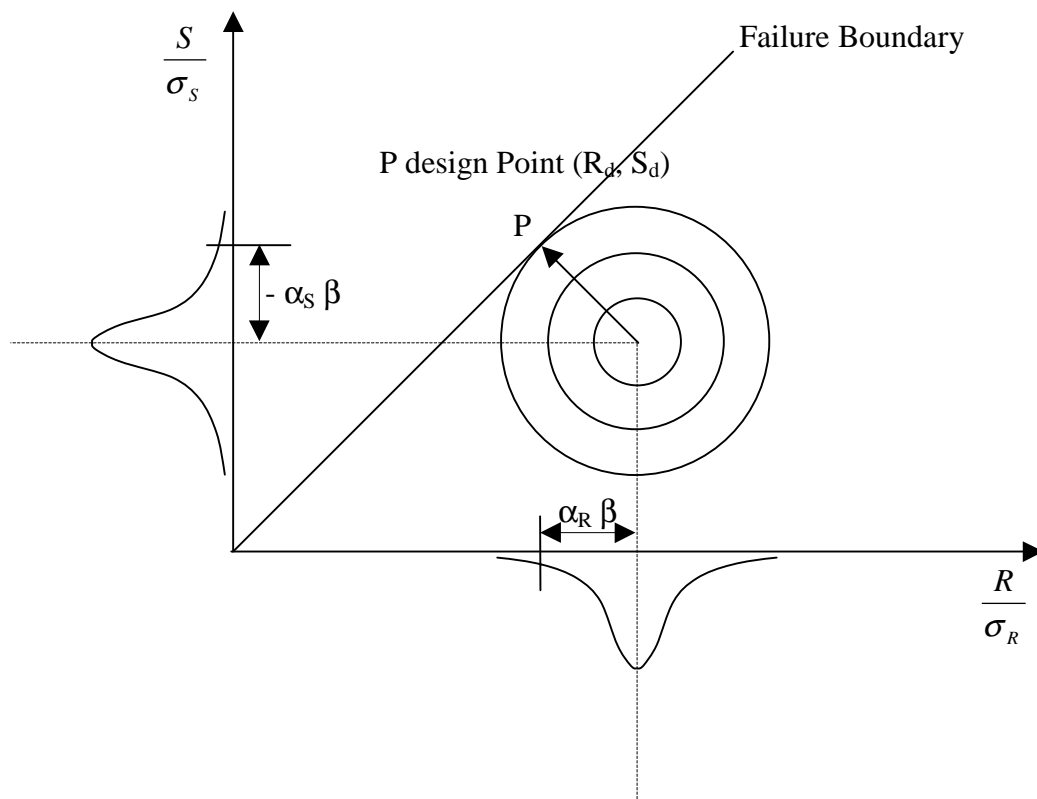


Figure 1: Design Point and reliability index β (Eurocode (2))

The design values of load effects S_d and Resistances R_d should be defined such that the probability of having a more unfavourable value is as follows:

$$P(S > S_d) = \Phi (+\alpha_S \beta) \quad (1a)$$

$$P(R < R_d) = \Phi (-\alpha_R \beta) \quad (1b)$$

Where β is the target reliability index; α_S and α_R , with $|\alpha| \leq 1$, are the values of the FORM sensitivity factors. The value of α is negative for unfavourable loads and load effects, and positive for resistances.

Eurocode (2) provides guidance of choosing α_S and α_R as -0.7 and 0.8 , respectively, provided that $0.16 < \sigma_S / \sigma_R < 7.6$, where σ_S and σ_R are the standard deviations of the load effect and resistance, respectively. This gives:

$$P(S > S_d) = \Phi (-0.7 \beta) \quad (2a)$$

$$P(R < R_d) = \Phi (-0.8 \beta) \quad (2b)$$

The Equation (2b) will be the basis for the analysis presented below.

3 SELECTION OF APPROPRIATE RELIABILITY INDEX

In Eurocode (2), the structures are grouped into three classes, namely, CC1, CC2 and CC3, based on the assumed consequences of failure and the exposure of the structure to hazards. The most appropriate class for bridges on major highways is the class CC3, which is described as “high consequences for loss of human life, or economic, social or environmental consequences very great”. The minimum reliability index recommended for CC3 class of structures is $\beta = 4.3$. Combining with Equation (5b), reliability analysis for yield line analysis should satisfy the following criteria for adequate safety: $P(R < R_d) = \Phi (-0.8 \times 4.3) = \Phi (-3.4)$.

Therefore the minimum reliability index for the yield line analysis will be

$$\beta_R = 3.4 \quad (3)$$

Since the variations in loads are not considered in the analysis, the appropriate load factors for loads need to be applied in the analysis. Appropriate statistical distributions will only be used for resistance variables. This method provides a rational basis to calibrate the partial safety factors involved in the resistance side of the equation, without having to determine the probability distribution of traffic loads, which is still a major unknown (5, 6 and 12).

4 PARTIAL SAFETY FACTORS

4.1 Resistance Factors

The partial safety factors incorporated in Austroads bridge design guide (1) are as follows:

- Bending moment capacity reduction factor, $\phi = 0.8$
- Use of characteristic strength of concrete and steel reinforcement. Characteristic strength value is usually defined as the 5-percentile value and is always lower than the mean value. The design strength values stated in the drawings are meant to be characteristic values.

According to the Austroads design guide (1), mean values are used for structural geometry, such as for depth of reinforcement and thickness of slab. This is less conservative than the stipulation by Eurocode (2), “Geometrical data shall be represented by their characteristic values”. The use of mean values instead of characteristic values for geometrical data and its implications on safety are also examined in the reliability analysis presented below in this paper.

4.2 Load Factors

The Bridge No. 1049 was analysed for T44 Truck loading. The following load factors as specified in Austroads bridge design guide (1) were used in the reliability analysis:

- Live Load Factor for the T44 Truck Wheel Loadings = 2
- Dynamic Load Factor = 1.25
- Multiple Lane Modification Factor for 2 lane loadings = 0.9
- Dead Loads = 1.4

5 RANDOM VARIABLES AND THEIR PROBABILITY DISTRIBUTIONS

The Table 1 presents the statistical parameters for random variables on the resistance side provided as guidance for reliability analysis in the CONTECVET (4) report. Similar values have been adopted by other research works presented in the literature (11, 14 and 7).

Table 1: Statistical parameters for Random Variables (4)

<i>Variable</i>	<i>Description</i>	<i>Distribution</i>	<i>COV (%)</i>
f_Y	Steel yield strength	Log-normal	5 – 10
f_c	Concrete strength	Log-normal	30 – 10
H	Geometry	Normal	5
ξ	Model error for flexure failure	Normal	5

5.1 Steel yield strength

The most sensitive parameter in yield line analysis is the steel yield strength. Therefore it is important that the statistical parameters for its distributions be estimated as accurate as possible.

According to the drawings, all deck reinforcing is cold worked deformed steel bars conforming to AS A83, A92 and A97. AS A83 specifies that cold worked deformed bars of size 3/8” or greater shall have a minimum yield or proof stress of 60,000 psi (~410 MPa) and a minimum ultimate stress of 70,000 psi (483 MPa).

Tensile tests were performed on a representative sample of 9 steel bar specimens. The mean and standard deviation of the yield strength were estimated to be 524 and 38.5 MPa, respectively. The coefficient of variation is 7.3%, which falls within the range of 5- 10 % guidance given by CONTECVET (4) report (Table 1).

In the reliability analysis, a mean value of 524 MPa was used for yield strength. The coefficients of variation of 5 % and 10 % were used to study the sensitivity of this parameter,

in addition to the 7.3 % found from the tests.. Yield strength is assumed to have lognormal distribution.

5.2 Concrete Strength

The drawings for the Bridge No. 1049 specify that all deck concrete shall have minimum strength of 4000 psi (28 MPa). A representative sample of six core specimens were taken for compressive testing. The mean and standard deviation of the sample are 32.5 and 8.15 MPa, respectively. The coefficient of variation is 25%, which falls within the 30 to 10% range given in Table 1.

For the reliability analysis, coefficients of variation of 10 and 30 % were used to study the sensitivity of this parameter, in addition to the 25% found from the test results. The mean concrete strength used was 32.5 MPa. Compressive strength of concrete is assumed to have a lognormal distribution.

5.3 Geometry

The CONTECVET (4) report (Table 1) guidance for coefficient of variation for geometry is 5%. For geometry, standard deviation is a more appropriate form to express the variations than COV, because depending on the reference point, position of reinforcement can be expressed with many different lengths, for example, as the distance from the bottom or distance from the top. The standard deviation should not, therefore, depend on how the distance is expressed. Other works reported in the literature (7), also support this notion and suggests the use of a standard deviation as absolute values (in mm or m) rather than a percentage of the mean length.

The slab thickness of 305 mm is the main geometrical parameter in this analysis. COV of 5% would give a standard deviation of 15 mm. This is similar to the value of 12 mm used by Holicky and Markova (7). Thoft-Christensen (16) used a standard deviation of 8 mm for covers with mean of 40 mm, a reasonable assumption. However, the COV in this case is 20%. This demonstrates the reason why the COV (%) is not the appropriate method of representation of geometrical variations.

In the reliability analysis presented below, the standard deviations of 10, 15 and 20 mm were used, regardless of the mean value of the dimension. The geometry/dimensions were assumed to be normally distributed, as stated in Table 1.

5.4 Model error for flexure failure

This allows for the uncertainties in the modelling process, due to many assumptions made (e.g. plane section remains plane, rectangular stress block, perfect steel/concrete bond and zero shear deformations). A coefficient of variation of 5% is applied to the calculated moment capacities, which are assumed to be normally distributed.

6 RELIABILITY ANALYSIS IN COBRAS AND ITS LIMITATIONS

Reliability analysis is an additional feature incorporated in COBRAS (3), which is a yield-line analysis software for flat slab bridges. The program takes the critical failure mechanism and load-case derived from the earlier deterministic collapse analysis and then undertakes a full

FORM (First Order Reliability Method) reliability analysis. Details of the FORM approach can be found in reliability analysis textbooks (10).

COBRAS (3) allows certain variables to be assigned as random variables, namely, Thickness of the slab (mm); Strength of concrete (MPa); Density of concrete (kN/m^3); Area of steel reinforcement (mm^2/m); Yield strength of steel (MPa); Depth of steel centroid to the global datum level (mm); Model uncertainty factor (incorporated into the reliability analysis using the membrane enhancement factor which acts as a multiplier on all moment capacities); and Intensity of loads.

Since it uses the FORM approach, the random variables can be assigned distributions other than *Normal Distribution*. The available distribution types in the software COBRAS (3) are *Normal*, *Lognormal*, or *Gumbel* distributions.

The reliability analysis performed by COBRAS (3) has two theoretical deficiencies, namely, it

- (1) does not include the effect the length of yield line on the variation of total moment capacities; and
- (2) performs the reliability analysis on the deterministically found minimum load factor mechanism, instead of finding the stochastically most relevant mechanism.

The two deficiencies mentioned above are elaborated in Sections 6.1 and 6.2 below.

6.1 Effect of the Length of Yield Line on the Standard Deviation of Moment Capacity

COBRAS (3) fails to take into account the effect of the length of yield line on the standard deviation of its moment capacity. For example, the standard deviation of the moment capacity is based on the standard deviation of the strength of the steel bar, among other random variables. The longer the yield line, the more the number of steel bars in the yield line. The variation (or the standard deviation) of the average of a large number of steel bars would be smaller than the variation of the average of strength of small number of bars. This is because the variation of each bar has a canceling out effect on each other when the average of a large number of bars is considered. A method describing how this effect can be considered in the analysis is described in a paper by Kowal and Sawczuk (8).

Since COBRAS (3) does not take this effect of the length of yield line into account, the variations considered in the software are higher than what they should be. However, this errs on the conservative side.

6.2 Stochastically Most Relevant Mechanism

COBRAS (3) determines the most relevant mechanism using deterministic analysis by minimising the live load factor. The mechanism and the results of this analysis are then used to calculate the reliability index, β . The mechanism determined by COBRAS (3) using deterministic analysis is not necessarily the same as the stochastically most relevant mechanism. The stochastically most relevant mechanism is the one with the smallest reliability index β , hence the mechanism with the highest probability of failure. A paper by Simoes (15) provides a methodology to calculate the stochastically most relevant mechanism

and demonstrates with a reinforced concrete slab example how the two mechanisms can be different (Figure 2).

The failure to consider the stochastically most relevant mechanism in COBRAS (3) errs on the un-conservative side.

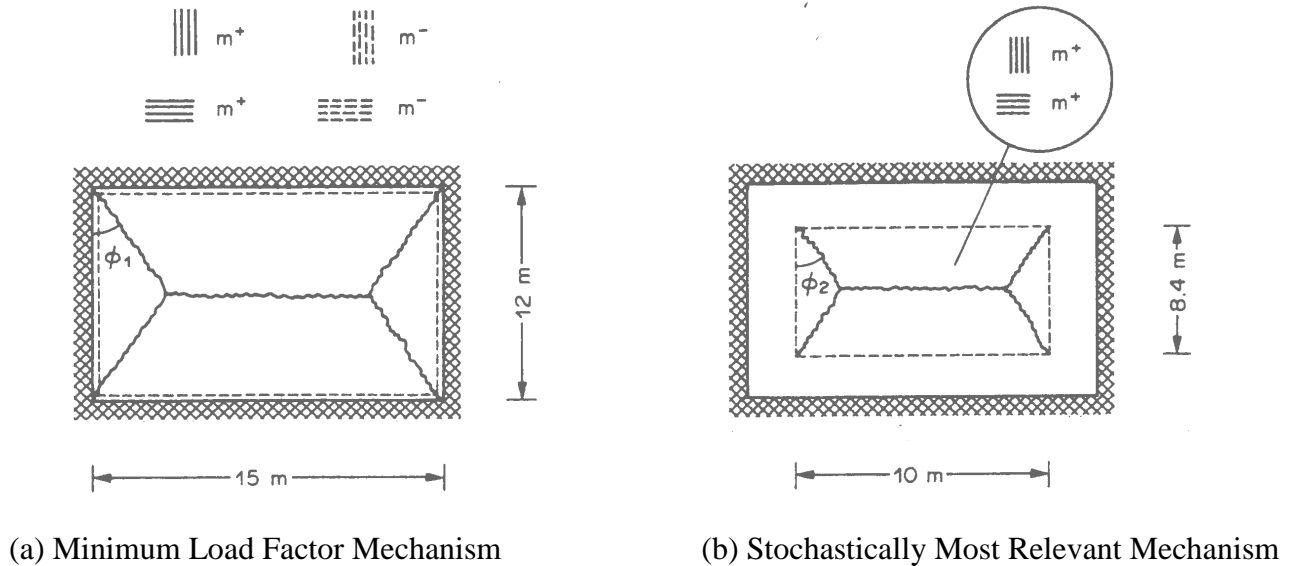


Figure 2: The Example from Simoes (15)

7 RESULTS OF RELIABILITY ANALYSIS

7.1 Load Rating with Nominal Values

In the current Australian practice of load ratings (1), the nominal values are used for strengths and mean values are used for geometry. The nominal values are the ones obtained from the drawings, and are usually expected to be the characteristic values. The nominal values used for load rating for the Bridge No. 1049 are as follows: Reinforcement strength, $f_y = 410$ MPa, 60,000psi(cf. characteristic value from the tests = 460 MPa); Concrete Strength, $f_c = 28$ MPa, 4,000psi (cf. characteristic value from the tests = 19 MPa); and Geometry = mean values of dimensions from drawings. A capacity reduction factor of $\phi = 0.8$ is applied to moment capacities.

The most critical condition was found to be in the end-span when two T44 trucks were present alongside each other. The critical yield line pattern found and used for the load rating is shown in Figure 3. The load factors used for the rating are as follows: Dead Load Factor = 1.4, for self-weight and superimposed dead loads; Dynamic Load Allowance = 1.25; Live Load Factor = 2.0; and Multiple Lane Presence = 0.9.

The load rating obtained from the yield-line analysis was 1.95 for this case. The 1.95 factor is in addition to the load factors mentioned above (1.4, 1.25, 2.0 and 0.9), i.e., the load rating of 1.0 would be the minimum for satisfying the safety criteria.

The reliability analysis was performed at this load rating. That is, the load factors applied for reliability analysis included $T44 \times 1.25 \times 2.0 \times 0.9 \times 1.95$ for live loads, and 1.4 for dead loads. The analysis was performed this way because at this level of live load, the yield-line analysis would give a load rating of 1.0; and the exercise was to find out the corresponding reliability index, β , at this load level. The parameters used for the analysis are presented in Table 2:

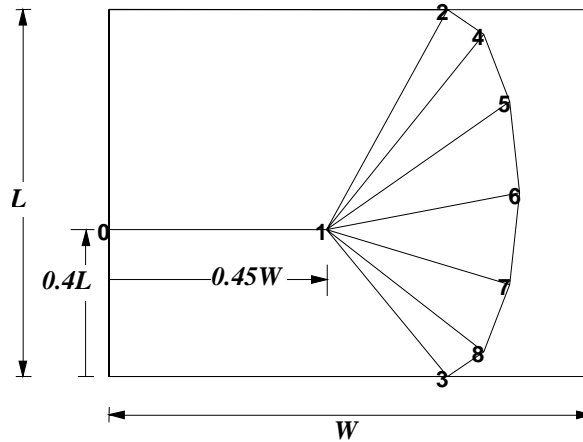


Figure 3 Position of nodes for worst-case geometry of critical mechanism

Table 2: Values used for Reliability Analysis at Load Rating of 1.95

Parameter	Distribution	Mean	Standard Deviation	COV
Steel yield strength, f_Y	Log-normal	524 MPa	38.5 MPa	7.3 %
Concrete strength, f_c	Log-normal	32.5 MPa	8.15 MPa	25 %
Geometry, H	Normal		15 mm	
Model error for flexure failure, ξ	Normal	1.0	0.05	5 %

In the reliability analysis, the capacity reduction factor, $\phi = 0.8$, for moment capacities was not used. Instead, a factor 1.0 was used with a standard deviation of 0.05. The mean values were used for strength and geometry values. The reliability index obtained from this analysis was $\beta = 4.40$. This is greater than the 3.4 mentioned earlier in this paper. This means the load rating of 1.95 corresponds to a $\beta = 4.40$.

7.2 Load Rating with Characteristic Values

In the previous section, the load rating was carried out using nominal values. The nominal values are the ones indicated in the drawings. They are supposed to be the characteristic values, but the nominal and characteristic values may not necessarily be the same, as stated in Section 7.1. The nominal value for steel strength was 410 MPa, compared to the 460 MPa of characteristic value found from the tests. In this case, the nominal value was more conservative than characteristic value, which is commonly the case for steel strength. This was not true for concrete strength. The nominal strength was 28 MPa, but characteristic strength was 19 MPa. This is a result of large variations in concrete test results, which is expected for cores taken from cast in-situ concrete.

Similar to the nominal values in Section 7.1, the reliability analysis is performed again for characteristic values. The characteristic values used for the load rating were as follows: Reinforcement strength, $f_y = 460$ MPa; Concrete Strength, $f_c = 19$ MPa; and Geometry = mean values of dimensions from drawings. The capacity reduction factor of $\phi = 0.8$ is applied to moment capacities.

The load rating obtained from the yield line analysis was **2.03** for this case, slightly higher than the 1.95 obtained for nominal values in Section 7.1.

The reliability analysis was performed at this load rating. That is, the load factors applied for reliability analysis included $T44 \times 1.25 \times 2.0 \times 0.9 \times \mathbf{2.03}$ for live loads, and 1.4 for dead loads. The parameters used for reliability analysis is same as those presented in Table 2. The only difference is the live load applied is slightly different from the previous one (2.03 instead of 1.95). The reliability analysis for this case resulted in a $\beta = 4.14$, slightly less for the increased loading, but still greater than the 3.4 benchmark value.

7.2.1 Use of Characteristic Values for Strength and Geometry

As mentioned earlier, in Eurocode (2), it is recommended to use characteristic values for geometry in addition to the characteristic values for strengths. This is not the recommendation by Austroads (1). However, the effect of using characteristic values for geometry is considered in the analysis presented below, which demonstrates that the use of characteristic values for geometry would significantly increase the safety and is reflected in the value of β .

The load rating was performed based on the values as follows: Reinforcement strength, $f_y = 460$ MPa; Concrete Strength, $f_c = 19$ MPa; and Geometry = characteristic values of dimensions, based on a standard deviation of 15 mm for all dimensions. A capacity reduction factor of $\phi = 0.8$ is applied to moment capacities.

The load rating obtained from the yield line analysis was **1.55** for this case, significantly lower than the 2.03 obtained above with the mean values of geometry.

The reliability analysis was performed at this load rating. That is, the load factors applied for reliability analysis included $T44 \times 1.25 \times 2.0 \times 0.9 \times \mathbf{1.55}$ for live load and 1.4 for dead loads. The parameters used for reliability analysis is same as those presented in Table 2. The only difference is the live load applied is significantly lower than the previous one (1.55 instead of 2.03). The reliability analysis for this case resulted in a $\beta = 5.88$, significantly higher due to the reduced load rating.

The results above demonstrate that by considering characteristic values for geometry, safety margin would be significantly increased.

7.3 Sensitivity Analysis

The difficulty with reliability analysis is estimating the correct standard deviations for each random variable. The following analysis shows the sensitivity of β to changes in each estimated standard deviations.

The following values were considered: Coefficient of Variation (COV) of steel: 5 and 10 %; COV of concrete: 10 and 30%; Standard deviation of Geometry: 10 and 20 mm.

The above combinations 2 x 2 x 2 resulted in 8 analyses. Further, a ninth analysis was performed using a standard deviation of 30 mm for geometry with COV of 10% and 30% for steel and concrete. The ninth analysis represented the worst-case scenario with very high variations in strength and geometry.

Each analysis was first performed to calculate the load rating based on characteristic values calculated as $mean \cdot (1 - 1.65 \cdot COV)$. The mean values were used for geometry. The live loads were then increased by this load rating, and the reliability analyses were performed. For the reliability analyses, same COV's assumed to calculate the characteristic values were used, along with the standard deviations of geometry. The reason for considering the standard deviations of geometry in the analysis is that the variations in geometry affect the β regardless whether it was considered in the load rating. The β values calculated based on these analyses are presented in Table 3 under Case (a).

The same analyses were then repeated to include the characteristic values for geometry. The results of these analyses are presented in Case (b) in the same table, Table 3. In these analyses, the characteristic values of dimensions as well as strength were considered for load ratings. Therefore, the load ratings were lower for the Case (b) load ratings. The β values calculated for Case (b) is same as for Case (a), except lower load ratings were used in Case (b). The β values found for Case (b) is significantly higher than Case (a), reflecting the lower ratings and increased safety margins.

Table 3 Summary of Reliability Indexes for Cases (a) and (b)

Analysis No.	Steel COV (%)	Concrete COV (%)	Geom SD (mm)	Case (a) Mean values of dimensions and Characteristic values of strength		Case (b) Characteristic values of dimensions and strength	
				Load Rating 2T44x1.25x2x0.9	β	Load Rating 2T44x1.25x2x0.9	β
Test	7.3	25	15	2.03	4.14	1.56	6.04
1	10	30	20	1.88	3.88	1.27	5.78
2	5	30	20	2.02	3.49	1.34	5.55
3	10	10	20	2.07	4.50	1.54	7.08
4	5	10	20	2.26	3.85	1.67	6.58
5	10	30	10	1.88	4.30	1.58	5.24
6	5	30	10	2.02	3.89	1.67	4.97
7	10	10	10	2.07	5.43	1.81	6.87
8	5	10	10	2.26	4.80	1.97	6.39
9	10	30	30	1.88	3.34	1.03	5.71

The Table 3 presents β values for various values of COV's and standard deviations within the practical limits. It shows that in all the cases, except one, the β values are greater than the 3.4 benchmark value. The only time the β value less than 3.4 was in the Analysis No. 9, where very high standard deviation for geometry, 30 mm, was considered.

Therefore, it is reasonable conclude that the load rating using characteristic values of strength combined with capacity reduction factor for moment capacities ($\phi=0.8$) provide adequate

safety margin, satisfying the benchmark level stated in Eurocode (2). However, when there are large variations in geometry, there is a reasonably strong argument to include characteristic values for geometry in load ratings.

8 CONCLUSIONS AND RECOMMENDATIONS

Load rating obtained using yield line analysis, based on characteristic values of strength but mean values of geometry, and including capacity reduction factor $\phi=0.8$, satisfy the benchmark safety index stated in Eurocode (2).

The results from this Case Study demonstrate that by considering characteristic values for geometry, the safety margin is significantly increased. Therefore, when a large variation in geometry is found or suspected in a particular bridge to be load rated, it is recommended to use characteristic values for geometry as an additional safety measure.

Characteristic values based on actual tests undertaken on a bridge (to be assessed) are seldom available. Nominal values stated in the drawings can be used with caution and should afford a tolerable reliability index. If available, it is likely that only a small number of tests will be used to calculate characteristic values. It would be sensible, and conservative, to use the lower value of the nominal values from drawings and characteristic values calculated from the tests.

The commercially available software, COBRAS (3), used for the analysis has some theoretical limitations, which makes the calculated reliability index approximate. However, it is felt that the errors due to these limitations are not very large.

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REFERENCES

1. Australian Bridge Design Code, Austroads Publications, AP-15, 1992.
2. BS EN 1990 (2002), "Eurocode – Basis of structural design", British Standards.
3. COBRAS, "CONcrete BRidge ASsessment", Version 1.7, Release 14, Middleton, C.R., United Kingdom.
4. CONTECVET, "A validated Users Manual for assessing the residual service life of concrete structures – A manual for assessing corrosion-affected concrete structures", EC Innovation Programme, IN30902I, GEOCISA, 2002.
5. Grundy, P., Khalaf, H., and Bouilly, G., "Direct Assessment of Aging Bridges – Traffic Loads", Austroads Bridges Conference, Adelaide, November 2000, pp. 147 – 157.

6. Grundy, P., Khalaf, H., Grundy, J., Taplin, G., and Bouilly, G., "Assessment of Structural Integrity of Bridges Using Weigh-in-Motion Data", First International Conference on Bridge Maintenance, Safety and Management, IABMAS 2002, Barcelona, July, 2002, 8 pages.
7. Holicky, M., and Markova, J., "Verification of Load Factors for Concrete Components Using Reliability and Optimisation Analysis – Background Documents for Implementing EUROCODES", Progress in Structural Engineering and Materials, Vol. 2, No. 4, 2000, pp. 502 – 507.
8. Kowal, Z., and Sawczuk, A., "On the Yield-Line Theory of Plates with Random Plastic Moments", Engineering Fracture Mechanics, Pergamon Press, 1976, Vol. 8, pp. 275-280.
9. Leicester, R.H., Pham, L., and Kleeman, P.W., "Use of Reliability Concepts in the Conversion of Codes to Limit States Design", Civil Engineering Transactions, The Institution of Engineers Australia, 1985.
10. Melchers, R. E., "Structural reliability: analysis and prediction", 2nd ed., Chichester : John Wiley, 1999, 437 p.
11. Nowak, A.S., Park, Chan-Hee, Casas, J.R., "Reliability analysis of prestressed concrete bridge girders: comparison of Eurocode, Spanish Norma IAP and AASHTO LRFD, Journal of Structural Safety, Vol. 23, 2001, pp. 331-344.
12. Nowak, A.S., and Szerszen, M.M., "Bridge Load and Resistance Models", Engineering Structures, Vol. 20, No. 11, 1998, pp. 985-990.
13. Pressley, J.S., Candy, C.C.E., Walton, B.L. and Sanjayan, J.G., "Destructive Load Testing of Bridge No. 1049 - Analyses, Predictions and Testing", Fifth Austroads Bridge Engineering Conference – Hobart, 2004.
14. Sørensen, J.D, Hansen, S.O., and Nielsen, T.A., "Calibration of Partial Safety Factors and Target Reliability Level in Danish Structural Codes", International Association of Bridge and Structural Engineering (IABSE) Conference on "Safety, Risk and Reliability – Trends in Engineering", Malto, 2001, pp. 179-184, pp. 1001 – 1006.
15. Simoes, L.M.C., "Reliability Assessment of Plastic Slabs", Computers & Structures, Vol. 35, No.6, pp. 689-703, 1990.
16. Thoft-Christensen, P., "Deterioration of Concrete Structures", First International Conference on Bridge Maintenance, Safety and Management, IABMAS 2002, Barcelona, July, 2002, 8 pages.