Footbridge Design for Synchronous Lateral Excitation

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SYNOPSIS

It is well known that longer span footbridges can feel lively to those walking over them. In general the acceptability of such liveliness is subjective, and depends on the individuals experiencing it, and the context. However it is now clear that there is an effect which is not subjective. It is called Synchronous Lateral Excitation (SLE), and when it occurs normal walking on the footbridge is disrupted.

Information on SLE was derived for the London Millennium Bridge (LMB) and is given in (1). The purpose of this paper is to show how a footbridge can be designed to take account of SLE.

For susceptible designs the relationship between the key parameters is studied in order to understand how the designs can be made non-susceptible. A distinction is made between elevated walkways, where most of the lateral flexibility is in the substructure, and long span footbridges, where most of the lateral flexibility is in the span. The former may have critical lateral frequencies even with short spans. SLE can occur in bridges of any span.

The inherent damping of modern footbridges is often very low, and designers will be surprised how many designs require additional dampers to meet the requirements of SLE. Configurations for dampers are discussed briefly.

1 SYNCHRONOUS LATERAL EXCITATION.

1.1 General Description

Synchronous Lateral Excitation is described in (1). This is the most comprehensive description currently available. It is hoped that there will be further research to collect data for a wider range of frequencies.

The publicity of the opening day of the London Millennium Bridge jogged memories, and information on previous events became available. Some of these are mentioned in the paper. It is relevant that the effect occurs not only in long span footbridges. It has occurred in short span footbridges, and also in a long span road bridge on a day when it was being used for a protest march. The common factor is that there has been a lateral mode with a frequency in the range of 0.5 Hz to 1.0 Hz.

Figure 4 of (1) shows lateral frequencies for a number of footbridges. It indicates that footbridges of any span can have a frequency in the critical range. The short span footbridges with a critical frequency may better be described as elevated walkways. Their lateral flexibility is derived from the slenderness of the substructures which support them.

There is a basic difference between human response to vertical excitation and to lateral excitation. Whereas the former is felt as a gradation of experiences as the dynamic magnitudes increase, the latter has a clear threshold beyond which lateral response of the bridge deck can increase rapidly and it soon becomes impossible to walk on it. This is SLE.

A walking person responds to the lateral motion of the bridge deck with small lateral forces which add energy to the system. As the lateral motion increases the lateral forces increase and also the proportion of people walking in phase with the deck motion also increases. The net effect is that the correlated lateral force per person can be expressed as kV_{local}, where V_{local} is the local lateral velocity of the deck, and k is a constant which depends on the frequency (see 1.2 below). When there are more than a critical number N_L of people walking on the deck the energy input exceeds the energy dissipated by damping, and the lateral motion increases relentlessly until the energy input reduces.

There is a further characteristic which became clear in the tests for the London Millennium Bridge. When the number of people was increased above N_L the instability did not always start immediately. It was as if there is some level of lateral movement below which the walking motions remain random. However if lateral motion is triggered by something, possibly wind, then SLE will be initiated. For design it should always be assumed that a trigger will exist.

The following expression for N_L is derived from Equation 9 in (1), with parameter μ introduced:

$$N_L = 4\pi cfM/\mu k \qquad \text{Equation 1}$$

where N_L is the limiting number of walking people on the deck
c is the damping coefficient expressed as the proportion of critical damping
f is the frequency of a lateral mode without any mass added for the pedestrian loading, in Hz.
M is the modal mass for the mode with frequency f, in kg
$\mu = \Sigma_{(i=1 to N)} \varphi_i^2/N$. For a sinusoid, μ=0.5. For constant φ_i, μ=1.0
φ_i is the lateral amplitude of the ith person, when the lateral amplitudes are normalised with the maximum modal amplitude of 1.0, as assumed in the computer analysis which calculated the mode shape
k is the lateral walking force coefficient. (See 1.2 below)

It is stated above that the frequency is calculated for the bridge without the mass of the pedestrians on it. There is some evidence for this, and this interpretation is conservative when calculating N_L. However it is unconservative when demonstrating that the frequency is above f_{MAX}. In these circumstances it would be prudent to consider also the frequency with one third of the design number N_L of pedestrians on it, calculated assuming each pedestrian has a mass of 75 kg.

It is tempting to ask at what amplitude the bridge deck goes critical? Unfortunately there is no answer to this question. If the number of people is close to N_L and, by some means, the amplitude is suddenly doubled, then the energy dissipated by damping is quadrupled and the energy input (the product of the force and velocity, which is proportional to the velocity squared) is also quadrupled. Hence the criticality (the ratio of energy input to energy dissipated) remains unchanged. The criticality is not related to the amplitude.

Is there a lower frequency limit beyond which lateral synchronous excitation does not occur? At lower frequencies there are two walking actions to consider. One is walking at a slow tempo, possibly a funeral march. The other is a snaking walk in which the walking tempo may be high, but the walker accommodates the lateral motion of the deck by following a snaking path which snakes with the frequency of the deck. The snaking walk is the natural behaviour for those walking on ships in a seaway. There is no lower limit to its frequency. The walking tests conducted for the London Millennium Bridge showed the characteristics of the snaking walk at 0.48 Hz, which was the lowest frequency studied.

1.2 Values for k

This paper is based on the tests made for the London Millennium Bridge, which were limited to the frequencies in the range 0.48 Hz to 1.03 Hz. Further tests will be needed to derive a k function which gives values of k to be used for f = 0 up to $f = f_{MAX}$, where f_{MAX} is the upper frequency limit beyond which it is deemed that lateral synchronous excitation will not occur.

For the present it is proposed that k = 300 Ns/m for all frequencies below f_{MAX}, and is zero for frequencies above f_{MAX}.

2 DESIGNING FOR LATERAL SYNCHRONOUS EXCITATION

2.1 Pedestrian Densities

What number of walking or marching people should the bridge be designed for? As the density of people on the deck increases, so it becomes less possible to walk or march freely, and the k value drops. There will be some value n_{MAX} of the number of people/m^2 which, when used with the full value of k, models the maximum energy input possible.

The following are reference values for $n_L = N_L /(L*B)$ Equation 2
where L and B are the length and breadth of the walkway, in metres.

Back analysis of LMB opening day event	n_L = 1.3-1.5	people/m^2
From studies made for the LMB retrofit	n_{MAX} = 1.6 to 1.8	people/m^2
LMB retrofit design basis	n_L = 2.0	people/m^2
IJburg Cycle Bridge design basis	n_L = 0.5	people/m^2

There is inadequate information on which to decide the design value for n_L. It is important that the client is brought into the process of selecting n_L. They may have to pay a significant additional sum of money for the bridge to achieve the design value. It will often be attractive to adopt a low value initially, with provision for additional damping to be added later if future research, or a full-scale trial, indicates it would be necessary

The LMB retrofit value is based on the assumption that n_{MAX} <= 2.0. This figure is believed to be conservative. The n_L value for the IJburg Cycle Bridge in Amsterdam was discussed with the client. It is low because the bridge is principally a cycle bridge and is not expected to carry pedestrians in large numbers. It is a suspension bridge with a main span of 170m. It has been designed with dampers in three locations to meet the n_L = 0.5 people/m^2 requirements. There would be disproportionate extra costs if this value were increased.

2.2 Values for f_{MAX}.

What is the highest frequency mode which can be excited by SLE? The answer relates to the behaviour of the people using the bridge. In normal use pedestrians are unlikely to be walking at more than about 2 Hz vertically, which excites 1 Hz laterally. An organised body of people could intentionally sustain a higher frequency. We have been informed by the Royal School of Military Music at Netherhall, UK, that the fastest marching unit in the British Army is the Light Division who march at 140 paces per minute, which excites 1.17 Hz laterally. The rest of the British Army march at 120 paces per minute (1.0 Hz). The value of f_{MAX} = 1.3 Hz given in (1) is a reasonable general design value. It should be increased if the designer knows of any particular uses which might require a higher value. In (2) an upper limit of 1.5 Hz is given, but we know of no basis for this figure.

2.3 Intrinsic Damping.

At the design stage it will always be necessary to check for SLE with only the intrinsic damping of the bridge, because the only reason for fitting damper units is because there is a concern that the bridge will not operate satisfactorily under intrinsic damping alone. It is not easy to find quantitative descriptions of intrinsic damping.

In general much of the damping in buildings and other structural systems comes from non-structural elements, such as partitions or finishes. A bridge has few non-structural components. There are usually only the parapets and a thin layer of deck finishes.

It may be assumed that the intrinsic damping is due to viscous behaviour within the structural material - or perhaps due to hysteretic behaviour. Some may be due to friction at interfaces either in the structure, or in non-structural elements. However SLE depends on the damping under small strains when much of the friction is not mobilised. The distinctions in behaviour are significant because each of these assumptions gives different answers when the question is asked "How should the design be modified to improve its SLE performance?"

For this paper it is assumed that, for a given type of structure, the damping ratio, c remains constant as the design is modified.

2.4 Design Theory

Once N_L has been chosen, Equation 1 should be applied to see if there is a possibility of SLE. If there is, it will be necessary to modify the design by modifying the damping, the stiffness or the mass.

To help the designer Equation 1 can be expressed in terms which reflect components of the design by expressing the frequency as:

$$f = 1/2\pi \ \sqrt{(K/M)} \qquad \text{Equation 3}$$

Where K is the modal stiffness.

So $$N_L = 2c^*\sqrt{(K^*M)}\ /\mu k \qquad \text{Equation 4}$$

This shows that N_L is proportional to the damping coefficient, and to the square root of both the mass and the stiffness.

It may seem counterintuitive to increase the mass to solve a dynamic problem. It should be recognised that, for a given frequency and velocity, increasing the mass increases the energy in the system, and the energy dissipated each cycle is a fixed proportion of this energy. However it should also be recognised that increasing the mass reduces the frequency. If this reduction takes it through the f_{MAX} limit then the increase in mass will introduce synchronous lateral excitation where none existed before.

2.5 Design of Elevated Walkways

The typical form of an elevated walkway is a deck structure with repeating spans which might be quite short, supported on piers which may be single columns to minimise interference with the activity below. Because of this configuration the M and K terms in Equation 4 are largely independent. The mass is mainly in the deck. The flexibility is in the piers.

To increase N_L it is necessary to increase either the mass or the stiffness, or both, but the benefit is won slowly because a 20% change is needed to achieve a 10% increase in N_L.

We are currently designing a scheme for a cycleway (150m long, 4m wide) across a tidal harbour in a region where any sizeable boats are excluded. The initial scheme shows precast prestressed beams spanning 17.5m between crossheads on single 1.2m diameter concrete filled steel tubular piles. The initial analysis shows a lateral frequency of 1.2 Hz and n_L = 0.9 people/m^2. It is clear that the design strategy will be to increase the stiffness of the substructure, and/or reduce the deck mass to ensure that all frequencies of lateral modes are comfortably above the 1.3 Hz limit. The alternative strategy, to increase n_L to, say, 1.8 people/m^2, would require the mass of the deck to be increased by a factor of four.

2.6 Design of Long Span Footbridges

In the design of long span footbridges the M and K terms are dependent. It is likely that most of the dead weight of the bridge is in the structure. Hence there is an approximate proportionality (or better) between the mass of structure and the N_L value. Any extra structure will be added in a way which benefits transverse stiffness disproportionately.

Long span footbridges are likely to be built in steel, with welded or tension-controlled bolted connections, and minimal finishes. In such bridges the intrinsic damping level will be very low. Text books quote various values for the intrinsic damping in such bridges but their values should be treated with caution. It is possible that much of the damping occurs around inclusions in the steel. In recent years steel manufacturers have improved their production methods in a number of ways - continuous casting, vacuum degassing, ladle stirring or heating - and so the damping levels may be reducing significantly.

A test on a recently constructed steel cable-stayed footbridge (with stone paving) showed c = 0.004 (3). The Steel Designer's Manual (4) quotes c = 0.003 for "unclad welded steel structures."

2.7 Damping Systems

The detailed specification of damping units is a specialist topic which is outside the scope of this paper. However it is necessary for the bridge designer to understand how a damper interacts with the structure, and to recognise the conditions in which a simple, cheap and reliable damper can operate effectively. For longer span footbridges it is quite likely that the needs of the dampers will dictate the configuration of the bridge.

Dampers act on the modal behaviour of the structure. If the damping level is quite low the mode shapes are not much affected. If a high level of damping is applied, the forces in the dampers will disrupt the behaviour and alternative modes will develop.

All dampers dissipate energy which is input to them through cyclic motion. Dampers can be used in a footbridge in three ways:

2.7.1 Internal Reactive Dampers

These are placed between parts of the structure which move relative to each other under the modal action which needs to be damped. In many forms of bridge construction, such as steel or concrete box girders, there are no significant movements between parts. Where there are relative movements they are usually very small and ancillary structure is needed to transfer the two sides of the movement to a location where the damper can act across a gap. This was one of the solutions used for the London Millennium Bridge.

In cable supported bridges there may be opportunities to exploit relative movements between the cables and the deck, or between different parts of the cable system.

2.7.2 External Reactive Dampers

There are likely to be large relative movements between the modal displacements of the deck and the surrounding landscape. The problem here is one of distance. It would be possible to attach a taut thin wire from the deck to an anchor structure on land. One end of the wire would be tensioned with a spring which has sufficient travel to follow the maximum displacement of the deck. The damper would be fixed across the ends of the spring. It would need to operate effectively under the small movements and forces which characterise bridge decks with stable crowds on them, and yet it must also be able to survive the much larger motions and forces imposed by high winds. It may be possible to design a damper unit with a bypass circuit which operates when the fluid reaches a certain pressure. The authors do not know of any systems like this.

It is a matter of semantics whether a damper which acts on the relative movements between the deck and the substructure is internal or external. We feel they are best described as external. Again there are examples on London Millennium Bridge. There are long telescopic cylinders placed diagonally between the top of the piers and the deck.

2.7.3 Tuned Mass Dampers (TMDs)

Where there is no relative movement, a single movement can be used in conjunction with the inertia of a mass. If the supports of the mass are tuned to the frequency of the mode to be damped, this can be effective with quite a small mass. However, if there is more than one problem mode, separate TMDs are needed for each mode. Low frequency modes require larger masses. TMDs have been used successfully on a wide range of structures. On a longer span footbridge with three or more modes to damp, it would require many dampers, and a significant additional mass on the bridge.

2.8 Limiting Pedestrian Movement

Footbridges are transport systems, designed for peak flows of pedestrians. They are also meeting places, and places where people can stop and admire the view. If the brief for the bridge has included some allowance in the deck width for dallying which is not required for transport, then the client may suffer unnecessary costs associated with the SLE measures needed for the additional width. In these circumstances partial barriers may be placed on the deck which limit the width of moving pedestrians to that necessary for peak flow requirements.

3 CONCLUSIONS.

The studies carried out for the retrofit of the London Millennium Bridge have provided a model of SLE which allows footbridge designs to be checked for this phenomenon. The studies also provide the data for the design of damping systems if added damping is needed. However, where the need is marginal, it will be difficult to decide whether to spend money on damping systems. For these bridges the designer needs to know the level of intrinsic damping with some degree of confidence, and this information is unlikely to be available.

Footbridge design will evolve in response to the requirements of SLE. The authors expect that additional lateral damping systems will become common, and much ingenuity will be demonstrated in developing bridge designs which can be damped without incurring too much additional cost.

4 REFERENCES.

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