Footbridge Pedestrian Vibration Limits

Part 1: Pedestrian Input

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Summary

The UK Highways Agency has commissioned two companion studies for the review of the dynamic sensitivity of footbridges. The aim of the research in this part of the project is to provide information on the dynamic properties of footbridges to enable designers to identify potential vibration susceptibility at the design stage. This will allow them to take appropriate action such as modify the design, or make provision for vibration absorbers. The work is being carried out by TRL Limited and Flint & Neill Partnership and involves a review of the main parameters affecting dynamic performance, the collation of data obtained from the vibration testing of footbridges, and the development of a design strategy. A separate project has also been commissioned by the Highways Agency to investigate the susceptibility of pedestrians to footbridge vibrations with a view to developing appropriate acceptance criteria. The results of this study form the basis for a new approach to the assessment of the dynamic responses of footbridges. It provides a much more searching analysis than the existing simplified approach, however it achieves this using methods that are not much more complicated to apply than the existing method. [1][2]

The paper presents an outline of the design strategy that has been developed, as a result of this project, for the prediction of vertical responses. It is anticipated that a procedure similar to that described in this paper will be developed for future inclusion in the Highways Agency's Design Manual for Roads and Bridges.

(This is one of a series of papers [3][4] *written to report on the progress of the above study. A companion paper* [5] *being presented at this conference deals with the somewhat subjective topic of bridge user perception and design limits.)*

Keywords: Footbridge; dynamic properties; pedestrian-induced vibrations; natural frequency; damping; stochastic analysis.

1. Introduction

The current practice in the calculation of footbridge responses, as defined by BD37/01 [6], is to measure the relative liveliness of a bridge by calculating the maximum acceleration that occurs when a standardised pedestrian crosses the bridge with a pace frequency that exactly matches the natural frequency(s) of the bridge. This approach can be shown to have a number of fairly significant shortcomings. These problems can be split broadly into 2 categories and are summarised below:-

Problems associated with the load model:

- The applied load does not represent pedestrian loading well; the 180N of the existing load model is significantly less than that which occurs in practice.
- The specific frequency ranges over which loads are most likely to excite a bridge are not considered at all. The test loading used by BD37/01 is always applied at the resonant frequency of the vertical mode under investigation. This is not what happens in reality where the spectrum of pedestrian loading from walking is centred strongly on 2Hz. Bridges with modes that are away from this frequency will be much less prone to excitation than similar modes at 2Hz, yet the present rules make no allowance for this, and are therefore unable

to differentiate properly between bridges that are likely to be excited by vertical footfall effects and those that are not.

- No attempt is made to calculate responses due to higher harmonics of the applied load.
- Other aspects of the load population are not represented at all.
- Bridges that are built for use in different circumstances will experience very different load populations, yet the current methods provide engineers with no means to deal with this.
- The relative magnitude, frequency and likelihood of walking versus running are not considered, nor is the likelihood and relative importance of responses due to single pedestrians compared with those in crowded conditions.

Problems associated with the response perception criteria:

- The response perception criterion used is highly simplified and does not agree well with the methods adopted in more modern British Standards.
- The duration of the calculated response is not considered. For example, two bridges that have the same peak response are treated as being equally 'lively' even if one is highly damped (where the motion stops quickly after the maximum is experienced) and the other is not (in which motion similar to that of the maximum amplitude might persist for some time).

In order to deal with the above in a more consistent manner two main changes are put into place.

1. We introduce the use of acceleration doses.

   Presently the acceptability of a bridge's response to a pedestrian crossing is only determined from the peak acceleration that occurs. Instead we choose to integrate the net discomfort caused over the full time history.

   BS6472 [7] introduces the notion of vibration doses. In this approach the fatiguing effect of a smaller but repeatedly occurring level of response has the potential to become more important than the effect of a single large instantaneous peak in the acceleration time history. The term 'relative discomfort' is used here to describe the relative discomfort likely to be experienced from two different response time histories and, by use of the aforementioned BS6472 approach, as the ratio of the 'root-root 4th power means' of the two histories.

2. Rather than considering the sensitivity of a bridge to a very particular (and potentially unrealistic) loading applied at the bridge frequency, we employ a stochastic model of the loading in order to investigate the likelihood of response, and apply this to a convolution integral approach where time history analyses of all possible cases are considered. (Probabilistic models of this sort have been suggested by a number of authors [8][9] but it is only relatively recently that computers have really been up to the task.)

   In addition the frequency dependency of pedestrian response perception has been modelled using ISO 2631 [10]. In this document design curves are presented that allow the calculated responses to be weighted and combined according to their frequency to adjust for this response perception.

This paper describes a revised calculation model that contains the basis for a new approach to the assessment of the dynamic responses of footbridges. It provides a much more searching analysis than the existing simplified approach, however it achieves this using methods that are, similar to, and not much more complicated to apply, than the existing method; and it is hoped that rules similar to these will replace the vertical response calculation rules of BD37/01in due course. Further details of the background to this method are due to be published elsewhere [1], and will be included in a TRL report likely to be published next year [2].

2. Proposed Codification Basis: A general method for deriving maximum vertical accelerations.

2.1 Introduction

The method for dealing with vertical accelerations uses results from a single time history analysis (for each mode) of a pedestrian crossing at resonance as a reference value which is then modified using a series of factors and combination processes that take account of the detailed dynamic properties of the bridge in question and variations in user perception.

Responses calculated using this method deal fully with multiple modes, harmonic load effects, pedestrian groups and crowd loads. However the method does not deal with responses where pedestrians deliberately walk in step, or the possibility that unexpected (non-random) vertical synchronisation might occur within crowds.

The dynamic responses of modes are split into 2 categories: responses near resonance that are calculated with respect to the above reference time history result; and quasi-steady responses that are calculated separately by direct integration of the mode shape.

2.2 Variables and notation

a_{eff}	Net effective vertical acceleration for the case considered (m/sec2)
$a_i(t)$	The instantaneous vertical acceleration of mode i at time t
C	A scaling constant used in the calibration of the calculated response
i	Mode index number
f	Mode frequency when expressed in cycles/sec (Hz)
f_i	Frequency of the ith mode (Hz)
f_{di}	Damped frequency of the ith mode (Hz)
$F(t)$	Amplitude of the fluctuating component of pedestrian loading at time t (N)
F_H	Harmonic H of the fluctuating pedestrian loading (N) – e.g. F_1, F_2, F_3
F_{qs}	Effective quasi-steady fluctuating pedestrian loading (N)
H	Harmonic number – of the pedestrian load model
M_i	Generalised mass of the ith mode
n	Number of modes used in response summation
N	Number of pedestrians in a group or on the bridge
N_{effi}	Effective number of pedestrians in mode i when they cover the whole span
s	Span length (m)
s_{effi}	Effective span length for mode i (m) defined as that length of a sinusoidally shaped mode that has the same maximum displacement and same enclosed area (displacement times span length) as the original mode shape.
t	Elapsed time (seconds)
V	Pedestrian velocity (m/sec) taken here as the mean speed of the population
x	Distance along span (m)
$g_i(x)$	Shape of the ith mode (consistent with generalised mass M_i above)
d	Structural damping (logarithmic decrement), $d = x.2.p$
k_{gi}	Relative discomfort factor dealing with effect of pedestrians groups in mode i
k_{popi}	Relative discomfort factor dealing with the effect of real populations in mode i
k_{spani}	Relative discomfort factor dealing with the effect of span in mode i
x	Structural damping ratio
$s_{dynimax}$	Dynamic component of standard deviation of response acceleration for reference case of mode i calculated at the mode maximum.
$s_{dynimax}(x)$	Dynamic component of standard deviation of response acceleration for reference case of mode i calculated at position x.
$s_{eff}(x)$	Net standard deviation of response acceleration at location x
s_{qsimax}	Quasi-steady component of standard deviation of response acceleration for reference case of mode i calculated at the mode maximum.
$s_{qsimax}(x)$	Quasi-steady component of standard deviation of response acceleration for reference case of mode i calculated at position x.
s_{refi}	Standard deviation of response acceleration for reference case of mode i
w_i	Frequency of the ith mode (radians/sec)

2.3 Contributing modes

Contributions to the dynamic response are to be considered for all modes below 5Hz, if there are no modes below 3Hz then responses should be calculated for modes up to 8Hz.

Because the points of maximum displacement (and therefore acceleration) occur at different locations along the span, normally responses will also need to be calculated at a number of points along the span.

2.4 Walking v running

Walking and running (including jogging) are treated as 2 completely separate load populations and therefore require 2 separate analyses to be undertaken. The following values should be used to represent these cases,

For walking:	For running/jogging:
$F_1 = 280$ N	$F_1 = 910$ N
$V = 1.7$ m/sec	$V = 2.0$ m/sec

The relative importance of walking and running for the structure under consideration is left to the designer, but both responses must be calculated and reported so that all parties are suitably informed about the responsiveness of the bridge being studied.

2.5 Calculation of pedestrian response

In general terms the process that needs to be followed in order to estimate the degree of discomfort likely to be experienced during pedestrian usage is as follows.

a) Calculate the reference response due to a hypothetical standard pedestrian crossing the bridge pacing in time with the natural frequency of each contributing mode.

b) Correct values of dynamic response to take into account realistic population parameters and thus allow for the likelihood of response occurring.

c) Calculate the quasi-steady (non-resonant) response that occurs. This term deals specifically with the responses that occur when the mode frequency and pedestrian population pace frequency are not very close.

d) Factor the component responses to allow for the number and distribution of the pedestrians being considered.

e) Combine the factored dynamic response and quasi-steady terms for each mode to determine the net response at each location of interest.

2.5.1 Calculating the reference dynamic response

For each contributing mode i and separately for walking and running cases, calculate the effect of a fluctuating dynamic point load, $F(t)$, moving across the span at a constant velocity, V (corresponding to the mean velocity of the pedestrians population), using:

$$F(t) = F_1.\sin(2\pi.f_{di}.t) \tag{1}$$

In which f_{di} is the damped frequency of the mode (Hz)

$$f_{di} = f_i\sqrt{1-\left(\frac{d}{2.p}\right)^2} \tag{2}$$

From the resulting response time history analysis calculate the reference standard deviation of the response acceleration. This may be achieved in one of a number of ways;

- For modes whose shapes are matched reasonably well by a simple half sine wave, values may be obtained directly from Figure 1 below.
- By the use of a suitable simplified algorithm. It is possible to calculate the contribution to a full response time

history from individual modes (of any shape) when loaded at resonance without recourse to expensive dynamic programs. The author has a simple Visual Basic algorithm (which can be made freely available) that does this task, and yet contains only 30 lines of code.

- By the use of an appropriate dynamic analysis program that has been modified to integrate discomfort as described above. Note, however, that the result required is the mode contribution to the response (at the point under consideration), not the net response from all modes that would be provided by many programs (though if used with care this may still be used without much loss of accuracy in many cases).

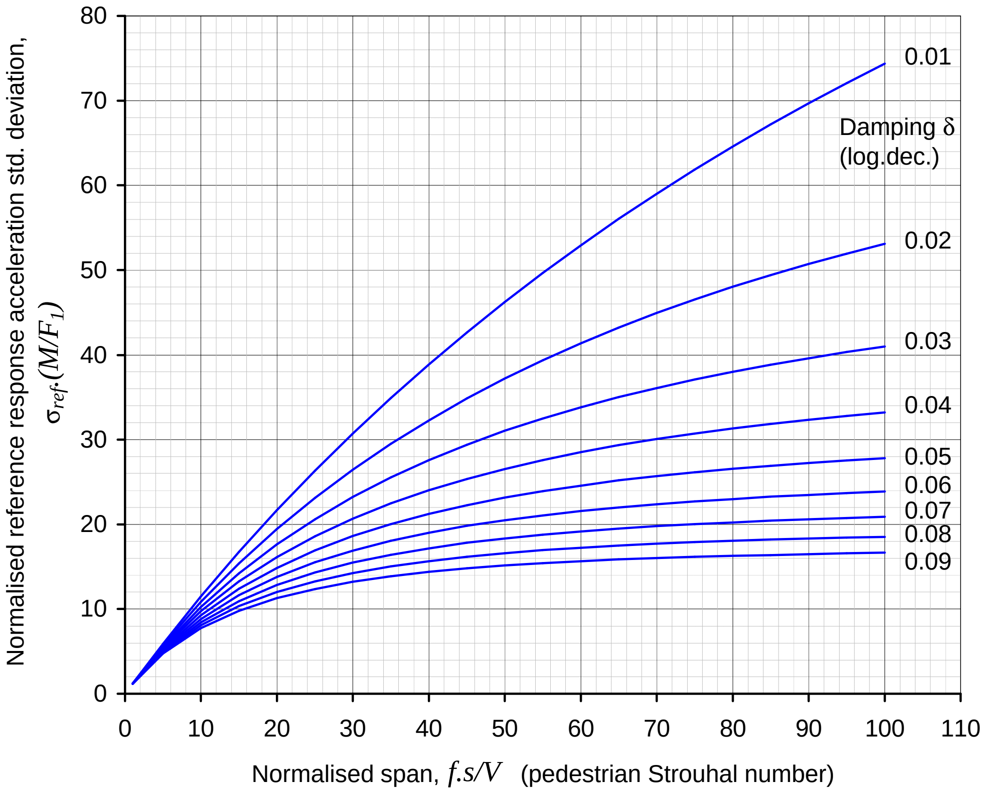

Figure 1: The contribution to the response of a simple span made by a pedestrian crossing with a pace rate that exactly matches the damped frequency of the mode (where the mode shape can be approximated by a simple half sine wave)

Thus for each mode of interest, determine the standard deviation of response for the reference case from Figure 1 above (if appropriate) or using the following formula:-

$$s_{refi} = \left[\frac{\int a_i(t)^2 .dt}{(s/V)} \right]^{0.5} \qquad (3)$$

2.5.2 Calculating the dynamic (resonant) contribution to the response

For each contributing mode of vibration, i, calculate the maximum amplitude of the dynamic (resonant) response for that mode from,

$$s_{dyni\max} = k_{spani} .k_{popi} .s_{refi} \qquad (4)$$

k_{spani} and k_{popi} are obtained from Figures 2 and 3 below. These factors modify the basic reference response case obtained above to allow for the effect that more realistic pedestrian population models have on response as well as to scale the response to include the relative variation of discomfort with frequency.

For each position, x, along the deck where responses are required calculate the mode dynamic contribution to the local vertical acceleration from,

$$s_{dyni}(x) = g_i(x).s_{dyni\max} \tag{5}$$

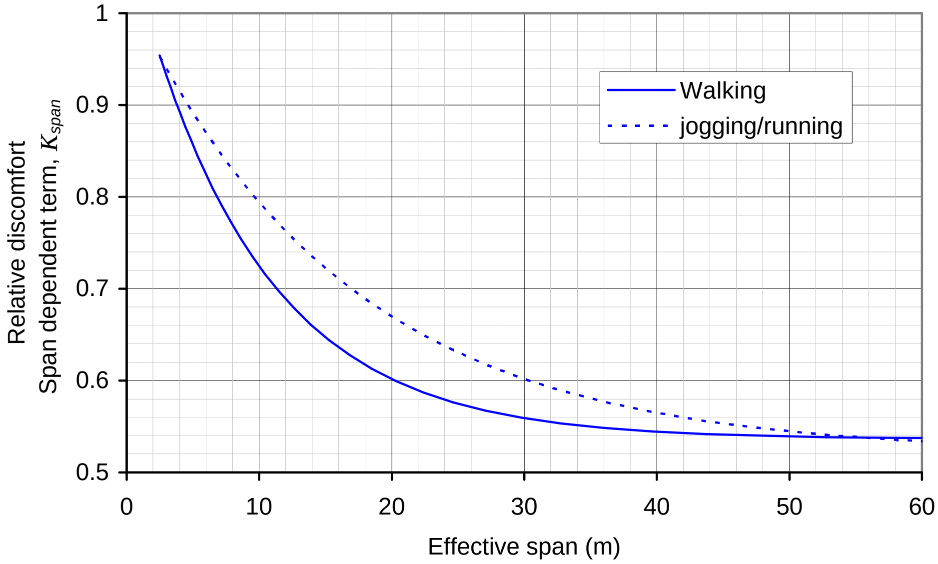

Figure 2: Dependency of relative discomfort on span length

Figure 2 uses an effective span length calculated for each mode, s_{effi}, obtained from,

$$s_{effi} = \frac{\int_0^s |g_i(x)|.dx}{0.637} \tag{6}$$

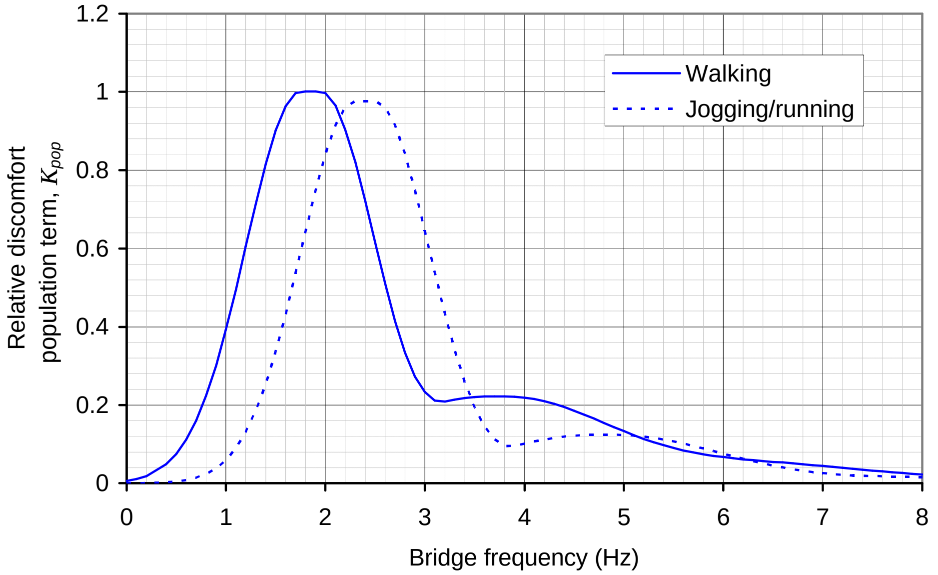

Figure 3: Dependency of relative discomfort on bridge frequency

2.5.3 Calculating the quasi-steady contribution of the applied load

For each contributing mode of vibration obtain the effective quasi-steady force from Figure 4 and the corresponding amplitude of the mode response from,

$$s_{qsi\max} = \frac{F_{qs}}{M_i} \cdot \left[\frac{\int_0^s g_i(x)^2 . dx}{2.s} \right]^{0.5} \tag{7}$$

For a simple sinusoidal mode shape this becomes

$$s_{qsi\max} = \frac{F_{qs}}{2.M_i} \tag{8}$$

For each position, x, along the deck where responses are required calculate the mode quasi-steady contribution to the local vertical acceleration from,

$$s_{qsi}(x) = g_i(x).s_{qsi\max} \tag{9}$$

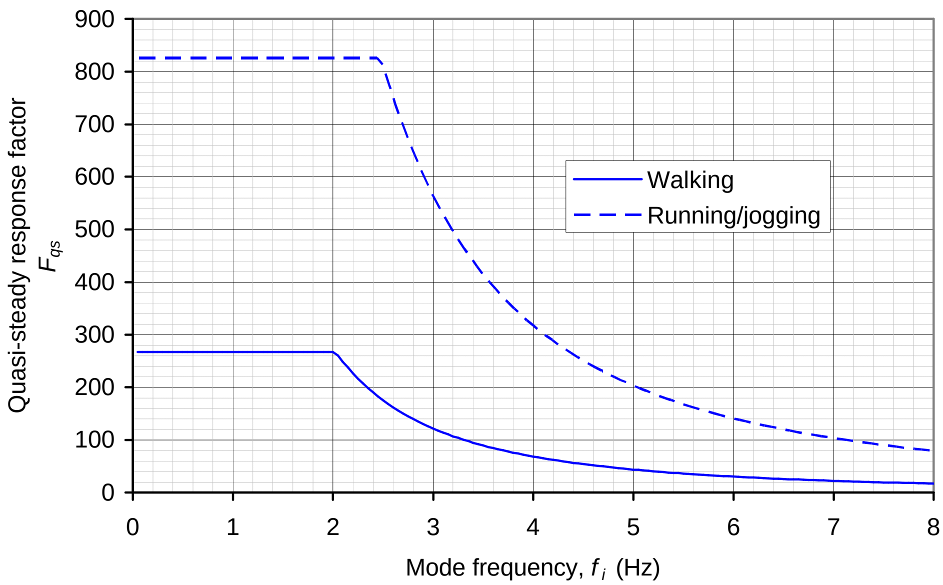

Figure 4: The effective quasi-steady force as a function of mode frequency

2.5.4 Combining component responses and factoring for pedestrian groups

The net standard deviation of response is obtained by combining the dynamic and quasi-steady terms using the formula below.

$$s_{eff}(x) = \left[\left(\sum_{i=1}^{n} k_{gi}.s_{qsi}(x) \right)^2 + \sum_{i=1}^{n} (k_{gi}.s_{dyni}(x))^2 \right]^{0.5} \tag{10}$$

Where k_{gi} is a factor, obtained from equations 11 and 12 below, that allows for the increase in discomfort that occurs when there is more than one pedestrian using the bridge at one time.

(It can be seen that the quasi-steady terms are summed using simple vector addition while the resonant terms are combined using a SRSS approach. However it should be recognised that for contributing modes that have very similar frequencies, the dynamic contributions of those modes should be combined with each other (in a more conservative manner) using vector addition. It is thought that a more general approach based on the complete quadratic combination method (CQC)[11] would be the most appropriate approach in complex situations, and particularly if significant numbers of modes are closely spaced.)

Pedestrian groups

For groups of N pedestrians crossing the span together in a single group, the likely degree of discomfort is increased in proportion to k_{gi}, where,

$$k_{gi} = 1.188\sqrt{N} \tag{11}$$

(This simplified formula provides a slight overestimate of response for very small groups, an alternative and more precise form of the relationship is given in section 4. Note that when $N = 1$, use $k_{gi} = 1$.)

Crowd loading

In crowded conditions or when pedestrians are distributed along the length of the span, the approach taken is based on superposition of the responses from single pedestrians.

The effective number of pedestrians considered to be active in each mode i is modified by the extent and shape of the mode and is given by the following equation,

$$N_{effi} = \frac{N.\int_0^s g_i(x).dx}{s.\max(|g_i(x)|)} \tag{12}$$

Where N is the total number of pedestrians on the bridge at any time.

Effective response used for assessment

The effective response acceleration to be compared with the comfort criteria (described in the companion paper) is given by,

$$a_{eff}(x) = 1.388 s_{eff}(x).(s/V)^{0.25} \tag{13}$$

This effective response defines comfort relative to the time taken for a single pedestrian to cross the bridge, and has been scaled so that it, for the same applied force, gives the same acceleration that would have been provided by a standard 50m bridge that just passed the requirements of BD37/01 (see section 3 for details).

3. Some background to the calculation of response

3.1 Reference responses

Throughout the time history, and for each point of interest on the deck, the accumulated degree of discomfort is to be integrated using the ‘root-root-4th power mean’ (RR4M) of the acceleration.

$$discomfort \propto \left[\int a_i(t)^4.dt\right]^{0.25} \tag{14}$$

However for simple mode shapes that can be approximated well by sine curves it can be demonstrated that,

$$\left[\frac{\int_0^T a_i(t)^4.dt}{T}\right]^{0.25} \approx 1.2\left[\frac{\int_0^T a_i(t)^2.dt}{T}\right]^{0.5} \tag{15}$$

In other words for simple cases the ‘root-root-4th power mean’ of the response acceleration is 1.2 times the ‘square-root-sum-of-squares’ (SRSS) of the calculated response. And even for more complex mode shapes this is usually not a bad approximation.

In the drafting of the above rules it was thought that;

- It would be advisable to present as many equations as possible in the simplest form that is practical, and
- It would be useful to provide the designer with a realistic measure of actual displacements rather than the somewhat abstract root-root 4th power mean.

It was therefore decided to create clauses that were based as much as possible on SRSS values and standard deviations of response rather than the RR4M values. In order to achieve this we simply require that when discomfort is integrated for specific mode shapes (that is if not using Figure 3.1) that s_{ref} is calculated using,

$$s_{refi} = \frac{1}{1.2}\left[\frac{\int a_i(t)^4.dt}{(s/V)}\right]^{0.25} \qquad (16)$$

This has the benefit of remaining consistent with the RR4M approach while having the convenience of working in terms of a notional standard deviation.

Consequently the equivalent acceleration used to define the discomfort experienced during a pedestrian crossing is correctly scaled from the measured discomfort of the reference response as follows,

$$\begin{aligned} a_{eff} &= 1.2C.s_{eff}.\left(\frac{s}{V}\right)^{0.25} \\ &\propto 1.2C.\left(\frac{1}{1.2}\left[\frac{\int a_i(t)^4.dt}{(s/V)}\right]^{0.25}\right)\left(\frac{s}{V}\right)^{0.25} \end{aligned} \qquad (17)$$

Where C is a calibration constant described below.

3.2 Calibration

In this document the calibration constant C is chosen so that, for a 50m span with d=0.03, a sinusoidal mode shape, that is known to just comply with BD37/01, using F = 180N (the fluctuating force used by BD37/01) these rules will give $a_{eff} = 0.5\sqrt{2}$ (that is the same response). However because this document uses a fluctuating force of 280N for walking the calculated response for a single pedestrian walking across the standard span would normally be (280/180) times that of BD37/01.

Note that a_{eff} is not a prediction of the maximum response, it is merely a measure, with units of acceleration, that is considered to be proportional to user discomfort. For other bridge configurations a_{eff} would not normally give the same result as BD37/01.

4. Conclusions

A number of recommendations have been made in order to improve predictions of structural response and user comfort to that response for footbridges. (Worked examples has been prepared to show the method by which the approach may be applied to a simple structure and can be made available on request. Simple Visual Basic functions are also available that reproduce all of the above figures but descriptions of these have been omitted from this paper to save space.)

This proposal has been based on the guidance given in ISO 2631 and BS6472 to provide a more relevant and modern basis that that currently given in BD37/01.

Although the analysis method proposed within this paper is based on the use of similar simplified analysis techniques as those used within BD37/01, it is able to deliver a much more consistent relative measure of response and discomfort for the wide range of structures that need to be assessed. While retaining much of the simplicity of BD37/01, the suggested approach is now also sufficiently general that it is able to cope with the complex mode patterns that are characteristic of many modern bridges.

When used in conjunction with the limits provided in the companion paper, the proposed method is likely to be more onerous, yet more realistic for high occupancy bridges and those with more sensitive users, particularly where bridge frequencies match the dominant frequencies of pedestrian walking and running. For bridges outside these ranges, the proposed codification should provide a more realistic set of criteria that match the lower sensitivities of such structures.

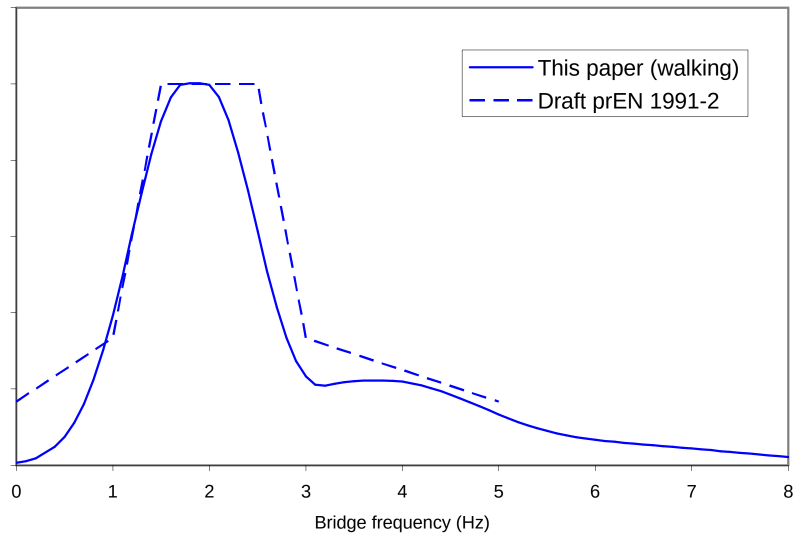

Figure 5: Predicted responsiveness compared with prEN 1991-2

Although derived in a different manner it is worth noting that the likely variation of responsiveness predicted by Figure 3 above bears a remarkable, but comforting, resemblance to the frequency sensitivity in the latest draft prEN 1991-2.

The authors would welcome comments arising from the use and applicability of the above proposals.

5. References

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